

# The Open-source Bayesian Atmospheric Radiative Transfer (BART) Code to Model Exoplanet Atmospheres

**Patricio Cubillos**<sup>1,2</sup>, J. Harrington, J. Blečić, P. Rojo, N. Lust, M. Stemm, R. Challener, A.J. Foster, A.S.D. Foster, S. Blumenthal, T. Loredó.

<sup>1</sup> University of Central Florida

<sup>2</sup> Space Research Institute, Austrian Academy of Sciences

**Exoplanetary Atmospheres and Habitability**

**Nice, France - 16/10/2015**





# Atmospheric Modeling:

- Radiative transfer. *Here!*
  - Cloud formation. *D. Homeier*
  - Magnetic fields. *R. Estrela*
  - Circulation dynamics.
  - Scattering.
  - Thermal equilibrium.
  - Photo-chemistry.
  - Vertical mixing.
- } *Grassi, Tsai, Blečić, Blumenthal,  
Hébrard, Venot, Hu*



# The Problem of Exoplanet Atmospheric Modeling:

- Low S/N observations.
- Photometry: few, broadband points. (e.g., Spitzer)
- Spectra: low resolution, span. (e.g., Hubble's WFC3)

Two approaches:

## **Forward Modeling:**

- Physically-motivated (+)
- Self-consistent (+)
- Non-exhaustive (–)
- Not necessarily unique solution (–)

## **Retrieval (e.g., Bayesian):**

- Statistically robust (+)
- Data driven (+)
- Computationally intensive (–)
- Not necessarily self-consistent (–)

Either way, **we need to know the physics well.**

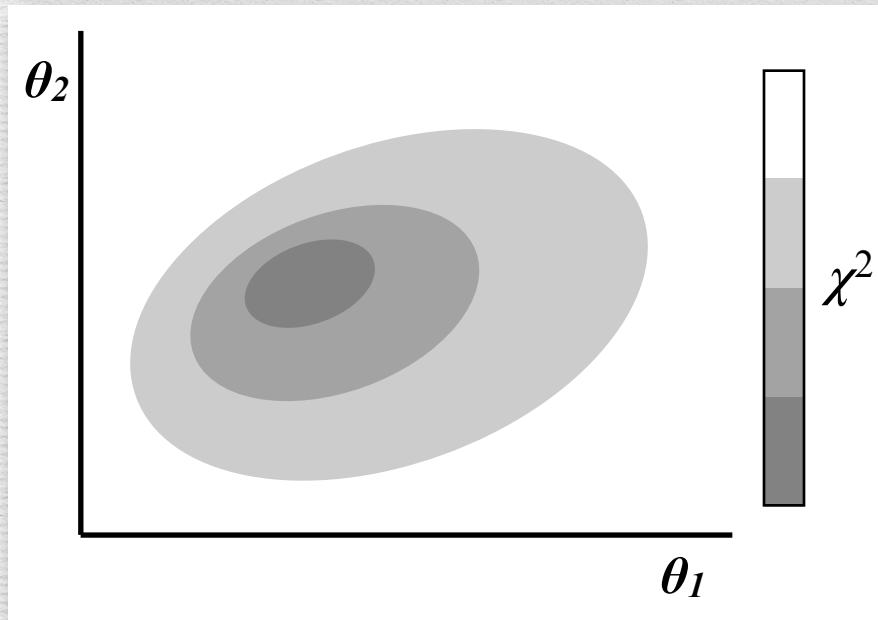


# Bayesian Statistics: Uncertainties estimation:

## Markov-chain Monte Carlo algorithm:

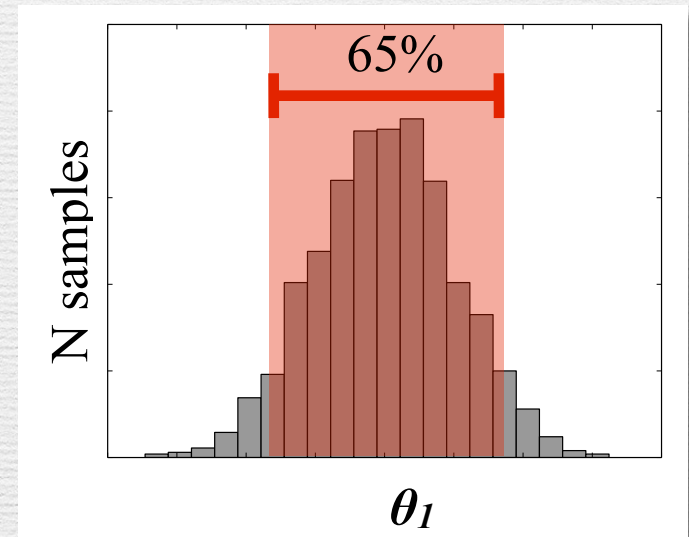
- Random sampling: Markov chain ( $10^5$ – $10^6$  samples).
- Accept/reject  $\propto$  goodness of fit ( $\chi^2$ ).
- Posterior distribution  $\propto$  parameter uncertainties.

## Posterior:



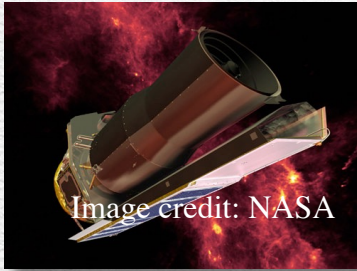
$$\chi^2(\theta_1, \theta_2 | \mathbf{d}) = \sum_i \frac{(m_i - d_i)^2}{\sigma_i^2}$$

## Marginal posterior:

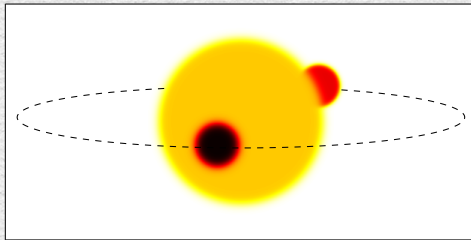


# Exoplanet Characterization:

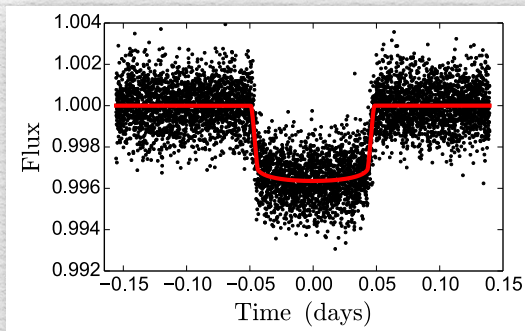
## Observations:



### Transiting systems:

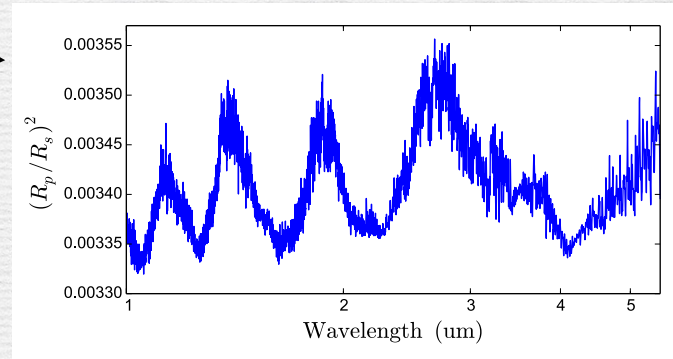


### Transit/eclipse depths:

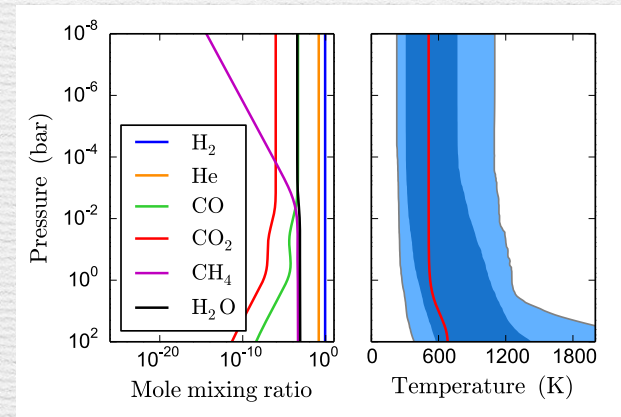


## Modeling:

### Transmission/emission spectra:



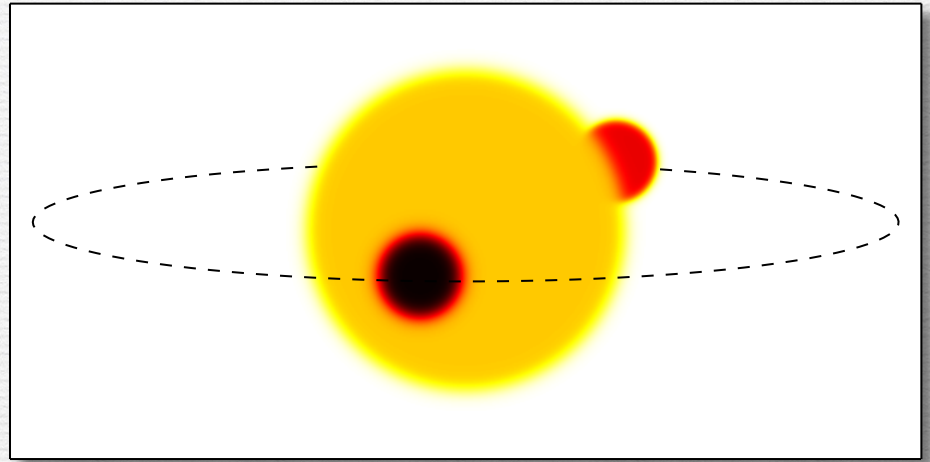
### Atmospheric characterization:



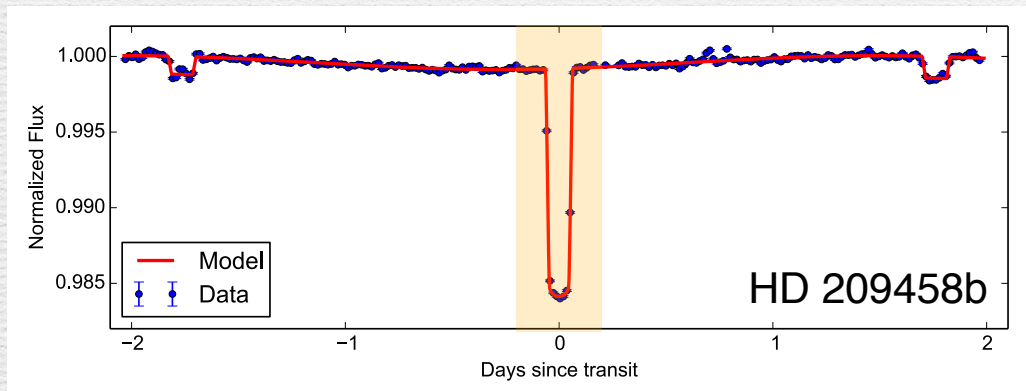


# Transiting Exoplanets:

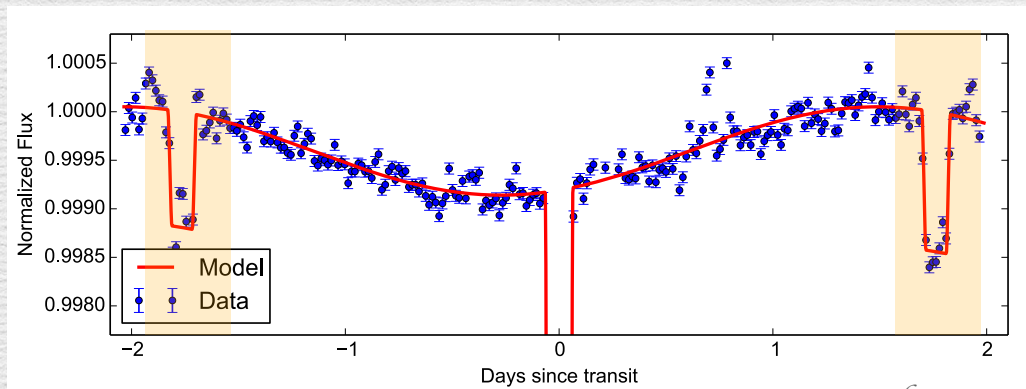
Planets that pass in front and behind their host star.



## IR Photometric time-series:



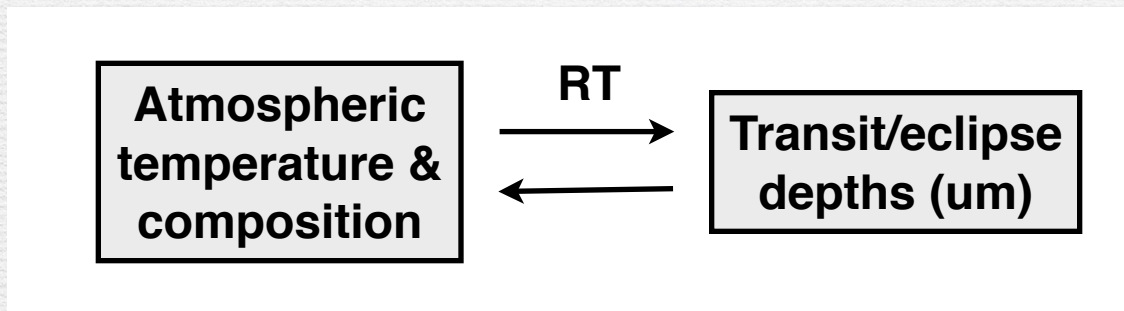
$$\text{Transit depth} = \frac{\Delta F}{F} \approx \left( \frac{R_p}{R_s} \right)^2$$



$$\text{Eclipse depth} = \frac{\Delta F}{F} \approx \frac{F_p}{F_s}$$

# Atmospheric Modeling:

The problem:



## Our solution:

- Radiative Transfer
- Thermo-chemical equilibrium
- (Bayesian) Statistics



## Radiative Transfer:

- Atmospheric model
- Species databases
- Observing geometry

## Atmospheric model:

- Temperature
- Species abundances

## Bayesian Atmospheric Radiative Transfer (BART) project.

Harrington, Cubillos, Blecic et al.

## Precursors:

Madhusudhan & Seager (2009),  
Line et al. (2013),  
Benneke & Seager (2012),  
Waldmann et al. (2015).



# Atmospheric Modeling:

## Bayesian Atmospheric Radiative Transfer (BART):

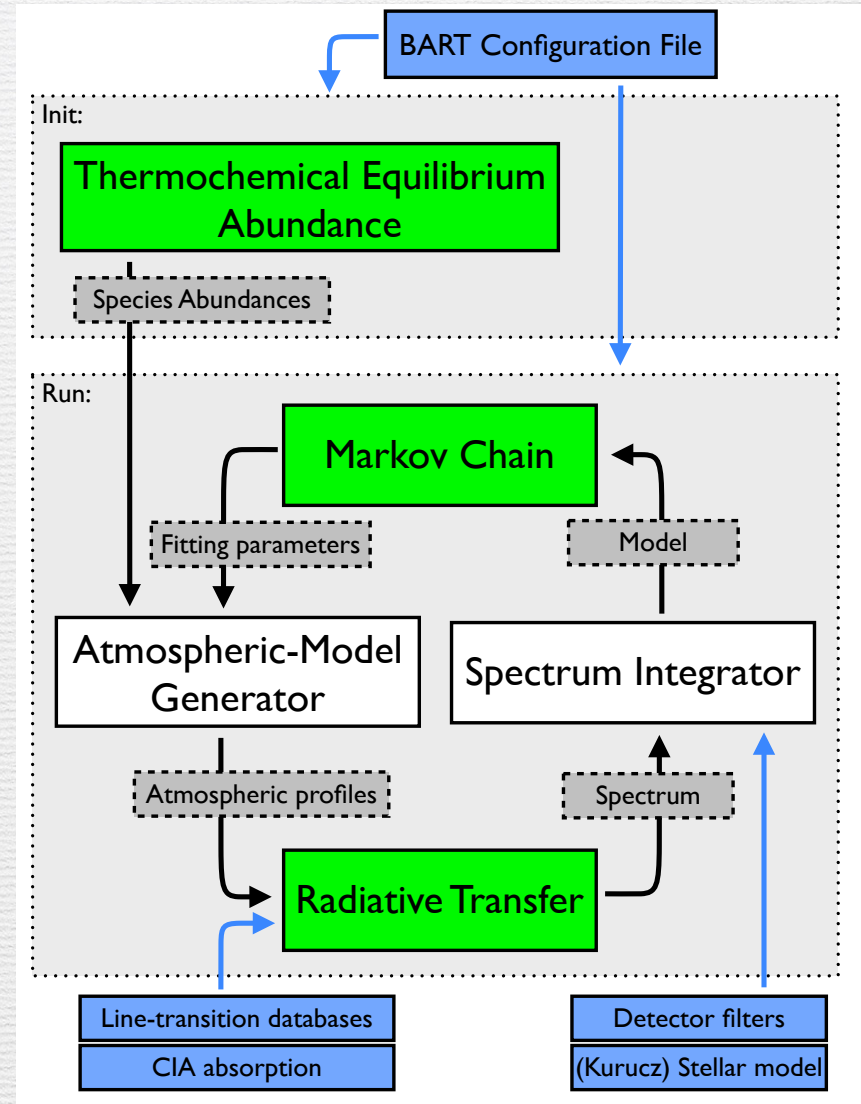
[github.com/exosports/BART](https://github.com/exosports/BART)



- Open-source Open-development.
- Documented.
- User interaction at the 'Python' level.



- Modular, independent, sub-modules:
  - Thermochemical-equilibrium chemistry.
  - Radiative-transfer.
  - Bayesian statistics.





# Radiative Transfer:

The **radiative-transfer equation** describes how light propagates, as it travels through a medium:

$$\frac{dI_\nu}{ds} = -\kappa_\nu (I_\nu - B_\nu(T))$$

path  $T, p, \text{composition}$

**Not really new science,**  
... but **must be efficient** (if MCMC).

- Each species has an specific absorption pattern in the spectrum.

## The Transit code:

- Solves 1D, line-by-line radiative-transfer equation.
- Produces transmission (transit) or emission (eclipse) spectra.
- Open-source, documented.

[github.com/exosports/transit](https://github.com/exosports/transit)

## Inputs:

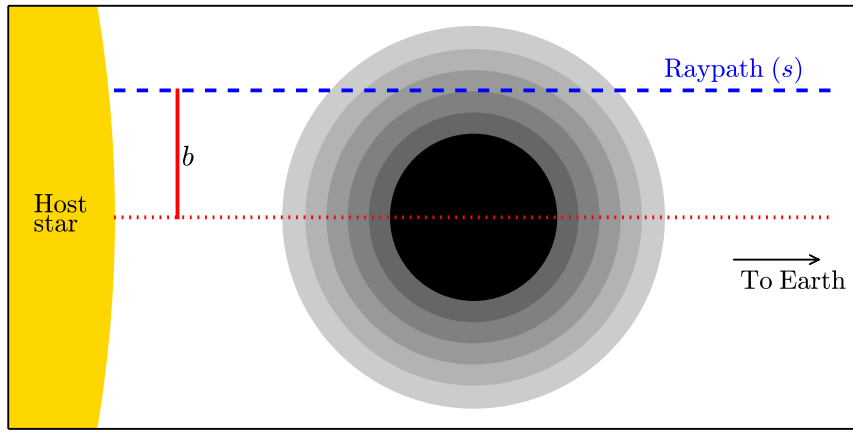
- One-dimensional atmospheric model ( $T, p, \text{abundances}$ ).
- Species opacity databases.





# Radiative Transfer:

## Transit Observing Geometry:



- 1D spherical-shells model
- Parallel rays

## Transmission spectrum:

$$\frac{dI_\nu}{ds} = -\kappa_\nu (I_\nu - B_\nu(T))$$

Transit depth

$$M_\nu = \frac{\Delta F}{F_s} = \frac{1}{R_s^2} \left[ R_{\text{top}}^2 - 2 \int_0^{R_{\text{top}}} \exp(-\tau_\nu) b db \right]$$

$$d\tau_\nu = \kappa_\nu ds$$



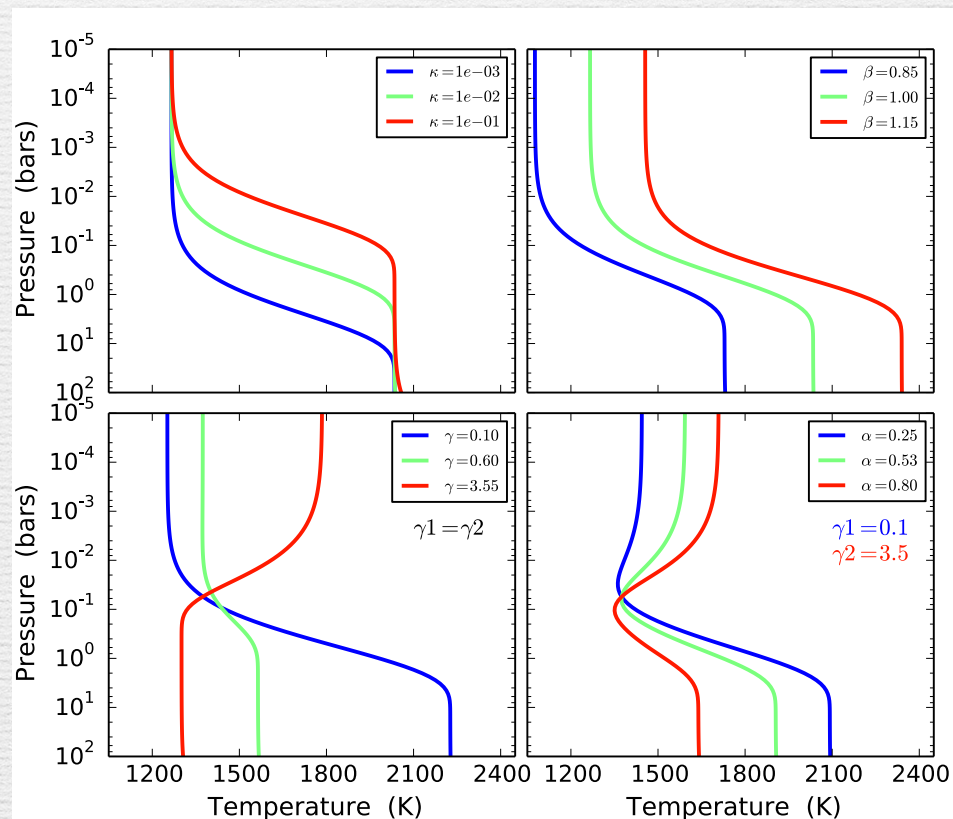
# Atmospheric Model:

## Temperature-profile model:

Three-stream Eddington approximation (Parmentier & Guillot 2014, Line et al. 2013):

$$T^4(p) = \frac{3T_{\text{int}}^4}{4} \left( \frac{2}{3} + \tau \right) + \frac{3T_{\text{irr}}^4}{4} (1 - \alpha) \xi_{\gamma_1}(\tau) + \frac{3T_{\text{irr}}^4}{4} \alpha \xi_{\gamma_2}(\tau)$$

- Stellar ( $T_{\text{irr}}$ ) and internal ( $T_{\text{int}}$ ) heating sources.
- IR ( $\kappa_{\text{IR}}$ ) and optical mean opacities ( $\kappa_{\text{v}}$ )





# Atmospheric Model:

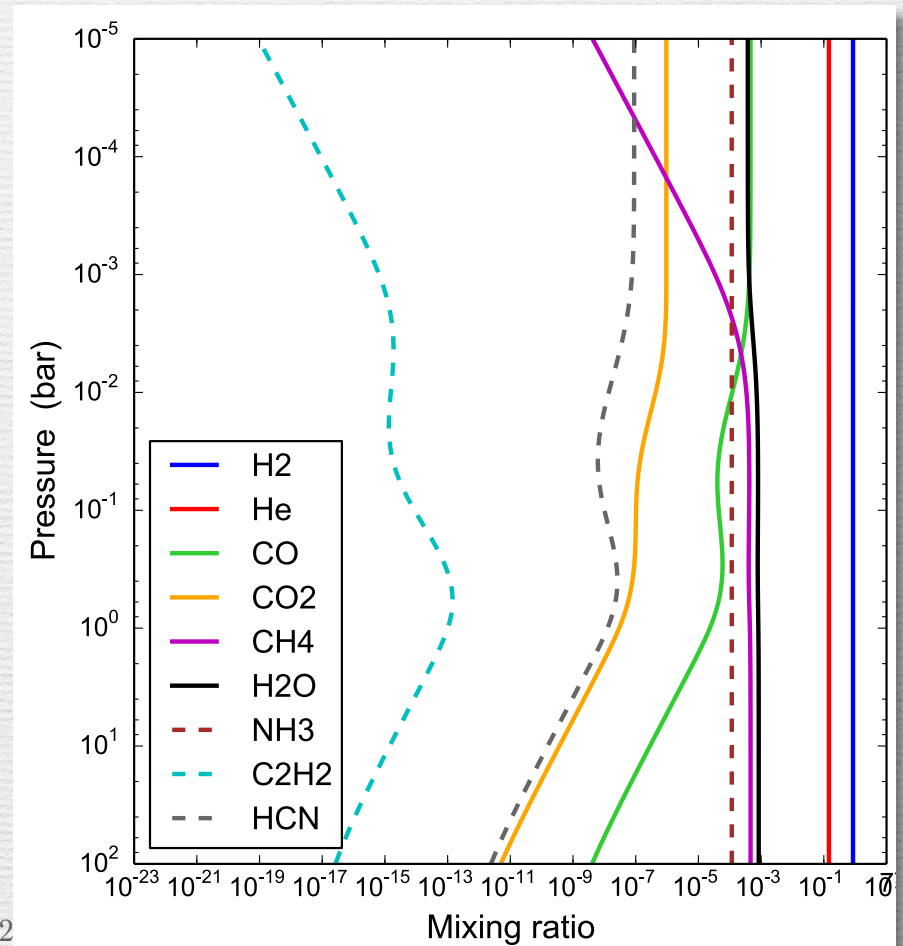
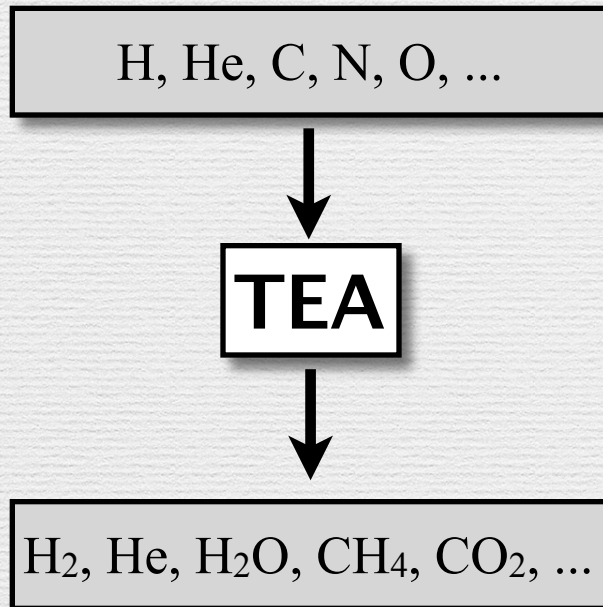
Thermochemical-Equilibrium Abundance (TEA) code: (Blecic et al. 2015a)

- Calculate species abundances (for given  $T$ ,  $p$ ).
- Python open-source code:



[github.com/dzesmin/TEA](https://github.com/dzesmin/TEA)

- Gibbs free energy minimization:



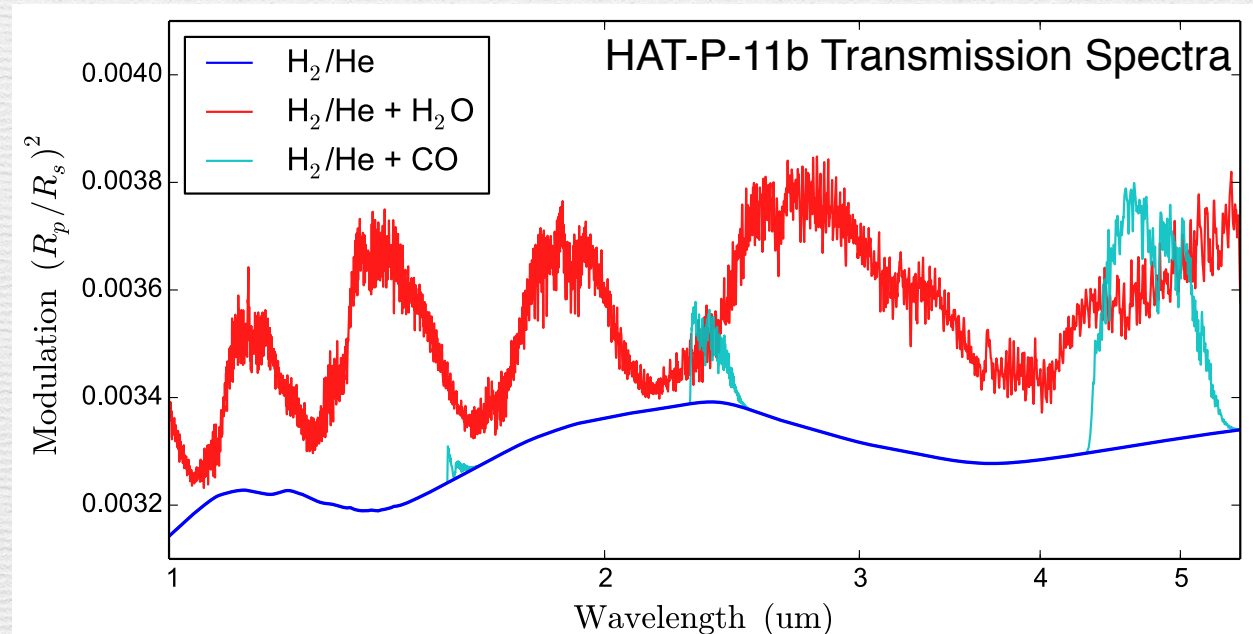


# Radiative Transfer:

## Opacity databases:

- Collision-induced absorption (CIA, [Borysow et al.](#), [Richard et al. 2012](#)):
  - Smooth variation.
  - Main species: H<sub>2</sub>, He.
- Quantum electronic, rotational, vibrational transitions ([Rothman et al. 2010, 2013](#)):
  - Discrete lines/bands.
  - Main species:

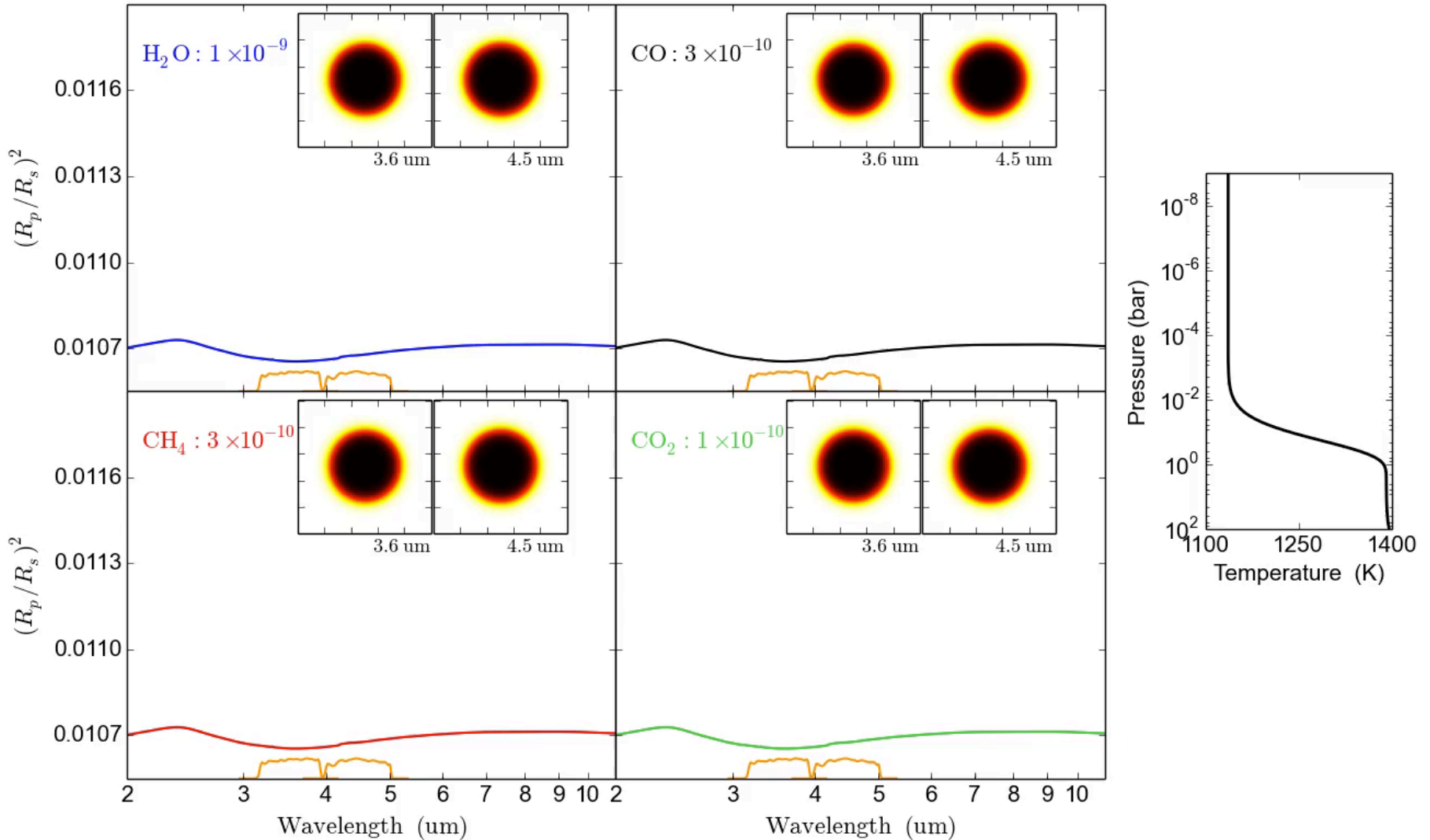
**H<sub>2</sub>O, CH<sub>4</sub>, CO, CO<sub>2</sub>,**  
**NH<sub>3</sub>, C<sub>2</sub>H<sub>2</sub>, C<sub>2</sub>H<sub>4</sub>, ...**





# Radiative Transfer:

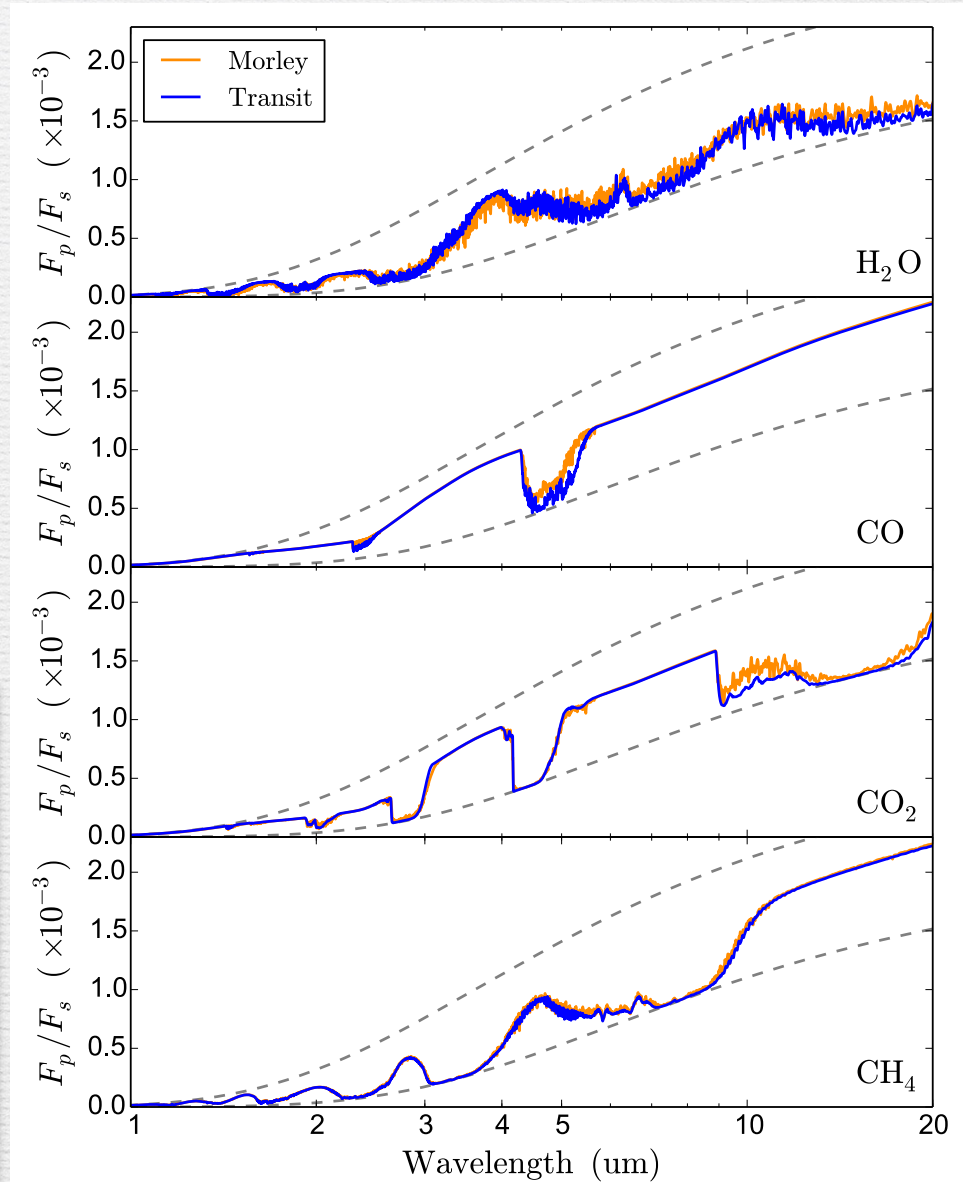
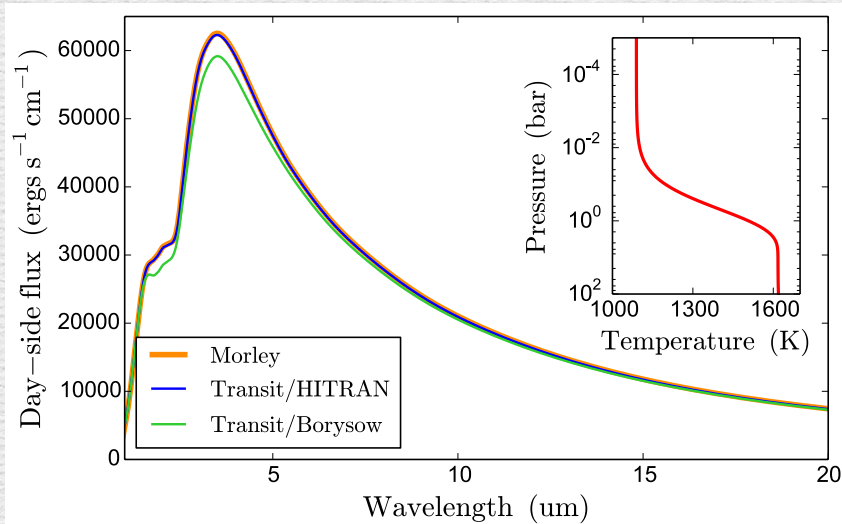
## Transmission spectra:





# Radiative Transfer: Validation

Our models agree to a few percent with those of C. Morley (in prep.).





# Statistical Package:

**Multi-Core Markov-chain Monte Carlo (MC<sup>3</sup>):** (Cubillos et al. 2015a, in prep.)

- **General** package for model-parameter estimation.
- Single/Parallel processor (MPI).
- Python/C open-source project:

[github.com/pcubillos/mccubed](https://github.com/pcubillos/mccubed)



- Differential-Evolution MCMC (Braak 2006).
- Gelman-Rubin MCMC convergence (Gelman & Rubin 1992).

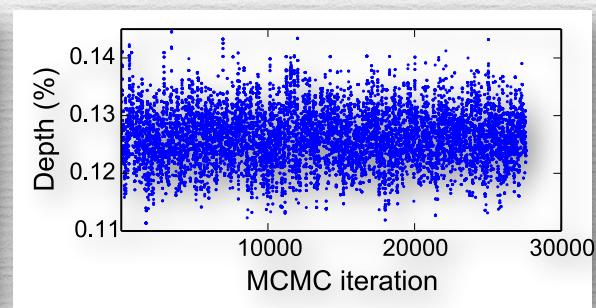
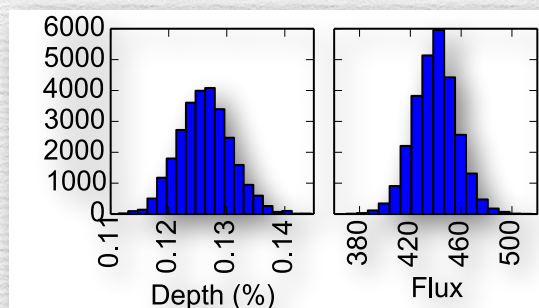
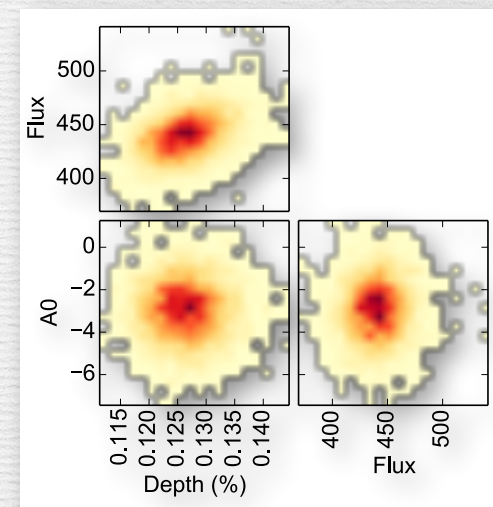
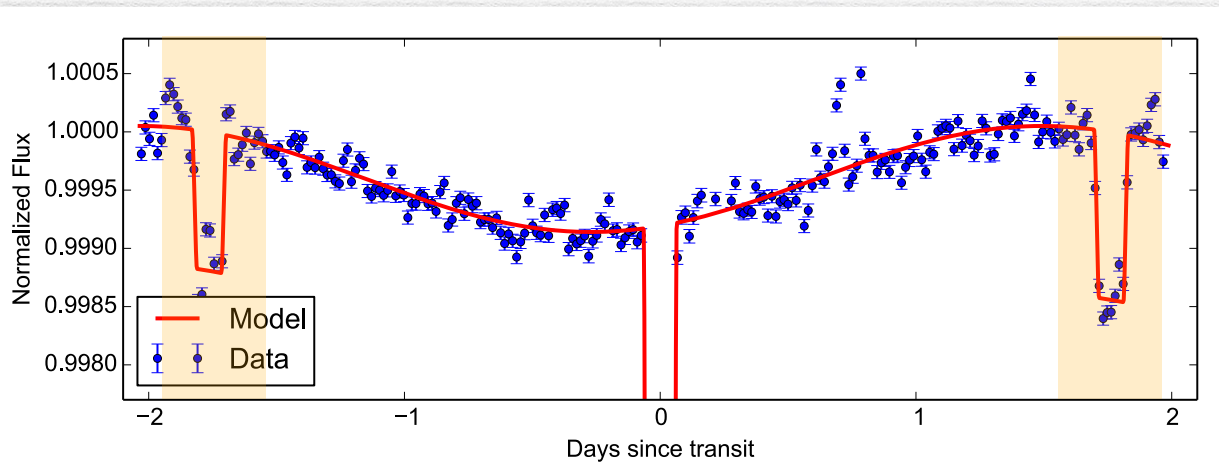


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Multi-Core Markov-chain Monte Carlo (MC<sup>3</sup>): [\(Cubillos et al. 2015a, in prep.\)](#)

[github.com/pcubillos/mccubed](https://github.com/pcubillos/mccubed)

HD 209458b Spitzer 4.5 um phase curve:

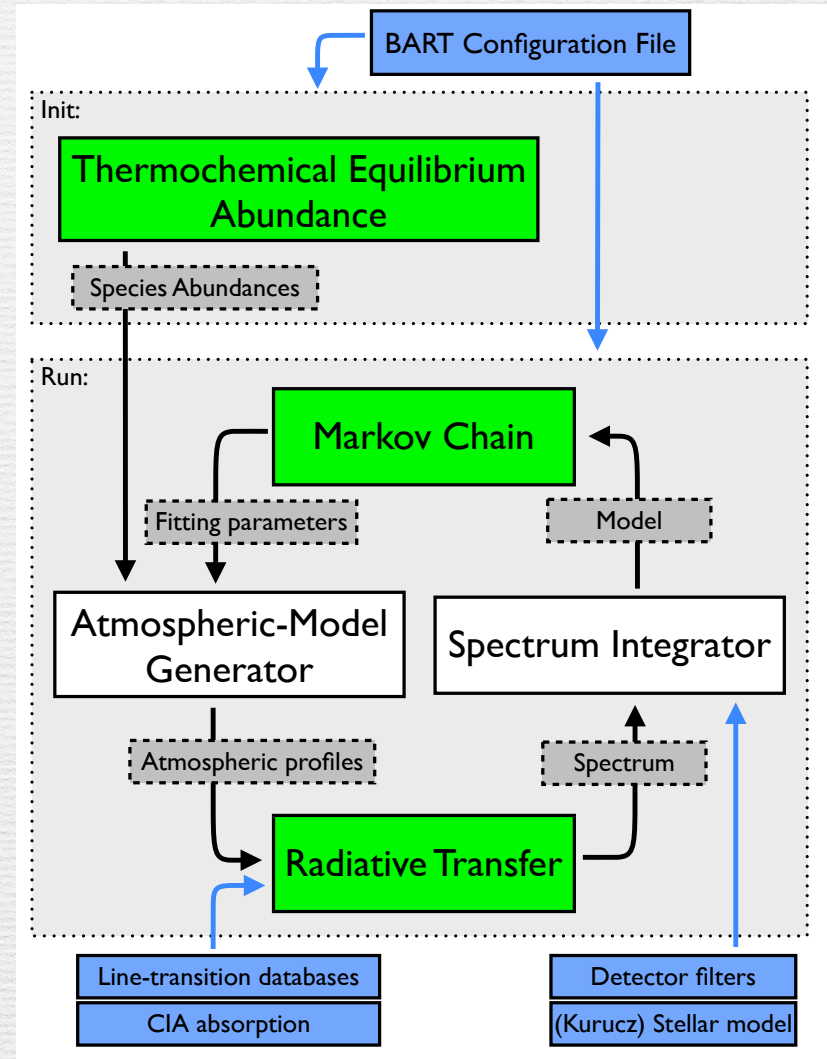
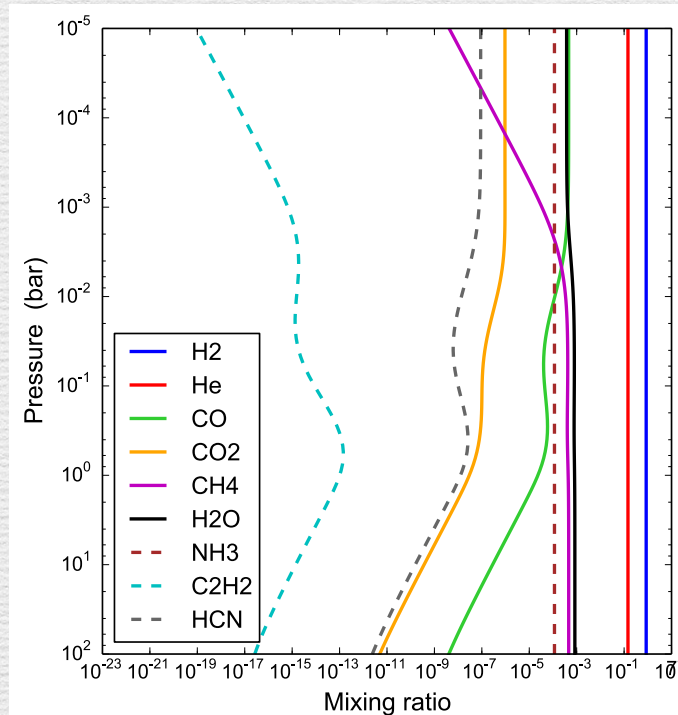




# Bayesian Atmospheric Radiative Transfer (BART):

## Free parameters:

- $T(p)$  parameters (3–5 parameters).
- Abundance parameters for each species:  
 $\log(f_{\text{H}_2\text{O}})$ ,  $\log(f_{\text{CH}_4})$ , ...
- Radius-pressure reference point:  
 $\text{rad}_0 = \text{rad}(p_0)$





# HAT-P-11b: Transit Observations

## Neptune-like planet:

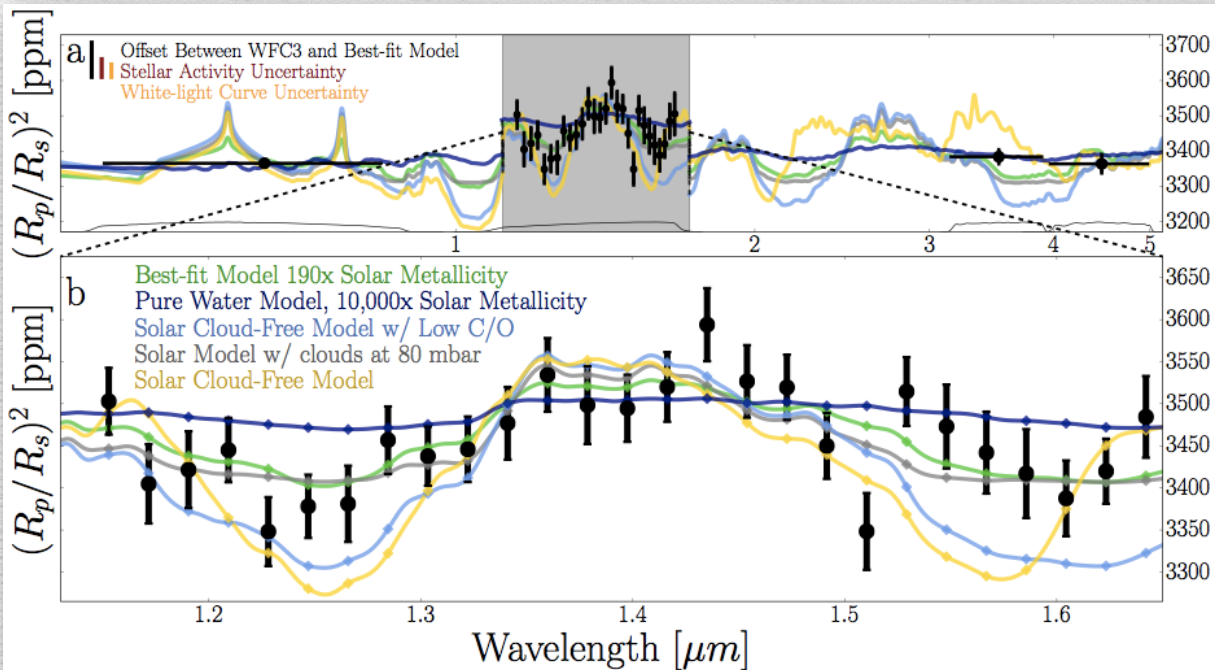
- $26 M_{\text{Earth}}$ ,  $4.7 R_{\text{Earth}}$ .
- 900 K equilibrium temperature.
- 5-day orbit period.

(Bakos et al. 2010)

## Transit observations:

- Kepler 0.64  $\mu\text{m}$ .
- Hubble's WFC3 1.1–1.7  $\mu\text{m}$ .
- Spitzer 3.6 & 4.5  $\mu\text{m}$ .

(Fraine et al. 2014)



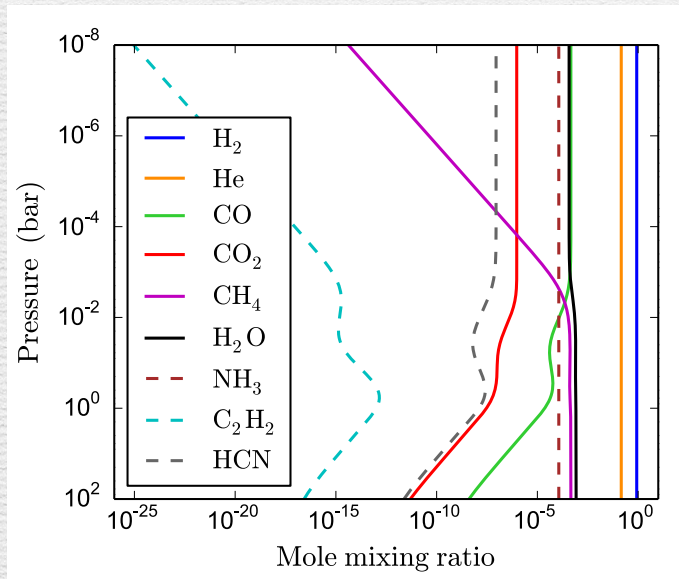
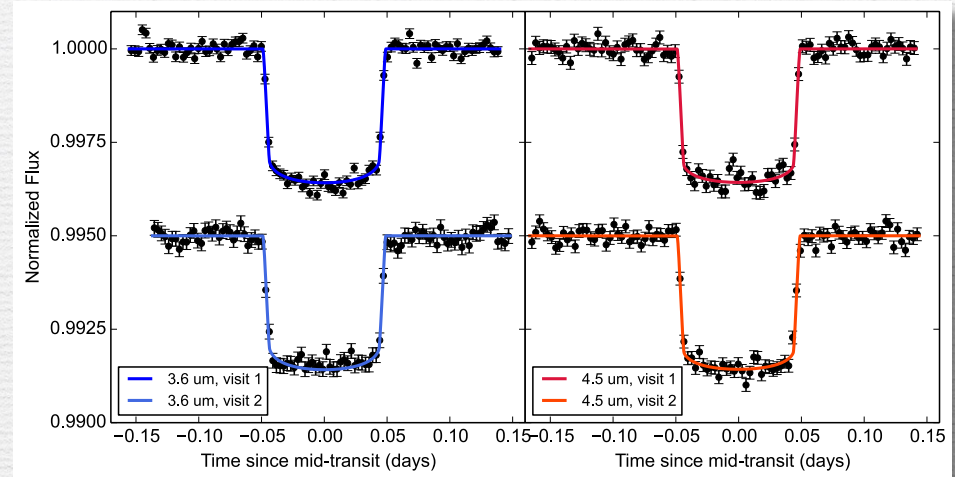
Atmospheric  
characterization with the  
SCARLET code:  
Benneke et al.



# HAT-P-11b: Transit Observations

Re-analyze Spitzer 3.6 and 4.5 um data:

Wavelength (um)	Transit depth	$R_p/R_s$
3.6	$0.00335 \pm 0.00002$	$0.05791 \pm 0.00022$
4.5	$0.00338 \pm 0.00003$	$0.05816 \pm 0.00025$



## BART fitting parameters:

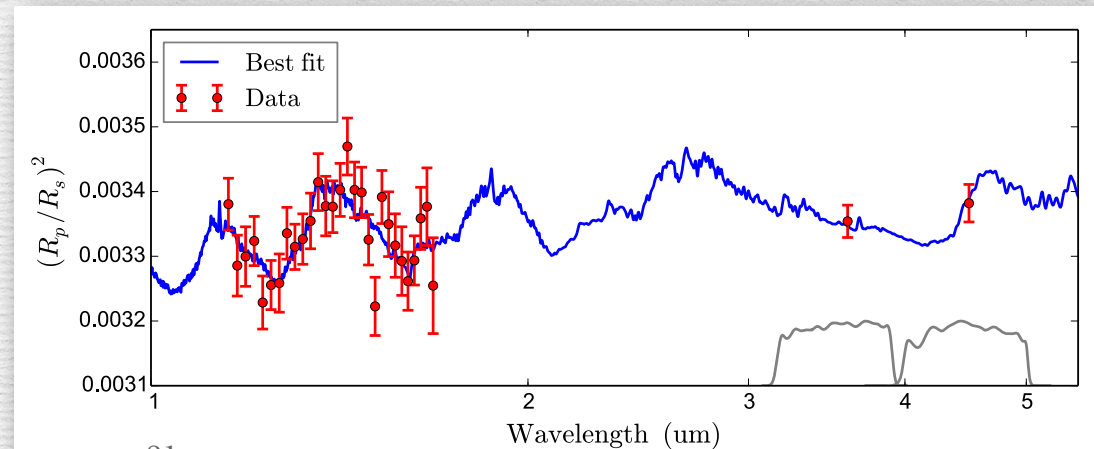
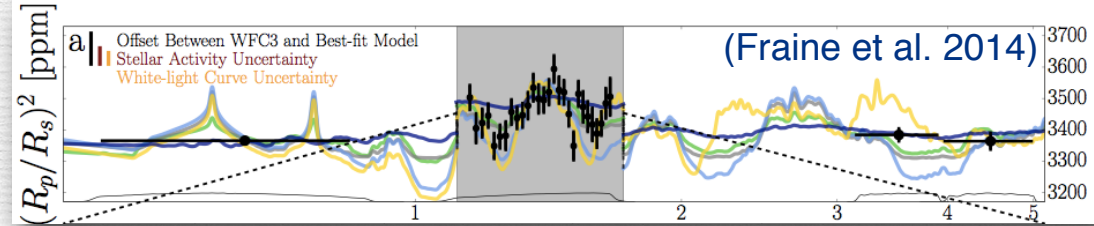
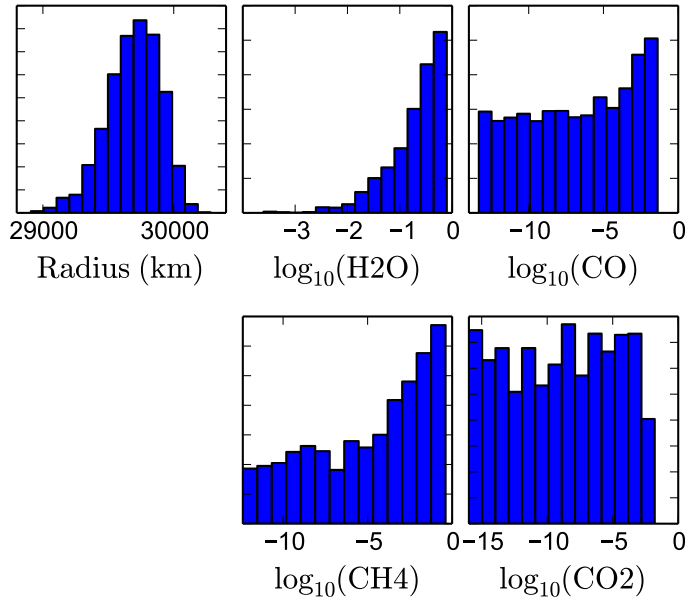
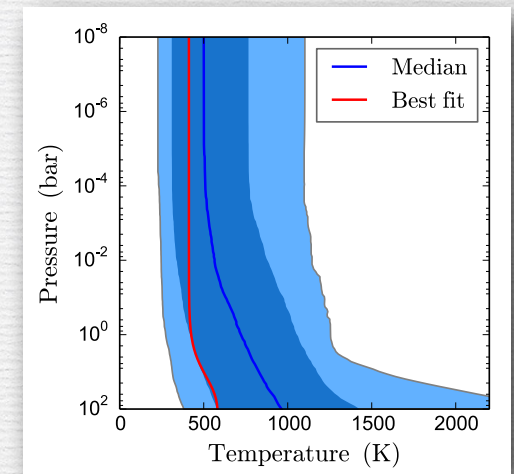
- H<sub>2</sub>O, CH<sub>4</sub>, CO, and CO<sub>2</sub> abundances **(4)**.
- Temperature-profile params **(3)**.
- Altitude at 0.1 bar **(1)**.

Started from equilibrium abundances.



# HAT-P-11b: Results

- Constrained the 0.1-bar radius to:  $29,750 \pm 200$  km.
- Determined  $\sim 100$  x super-solar abundance of  $\text{H}_2\text{O}$ .
- $\text{CH}_4$ ,  $\text{CO}$ ,  $\text{CO}_2$  remained unconstrained.

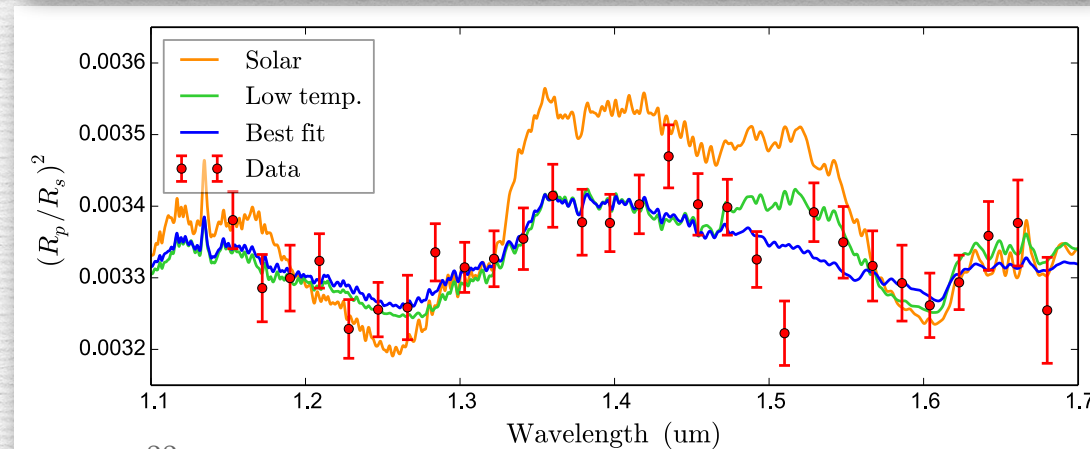
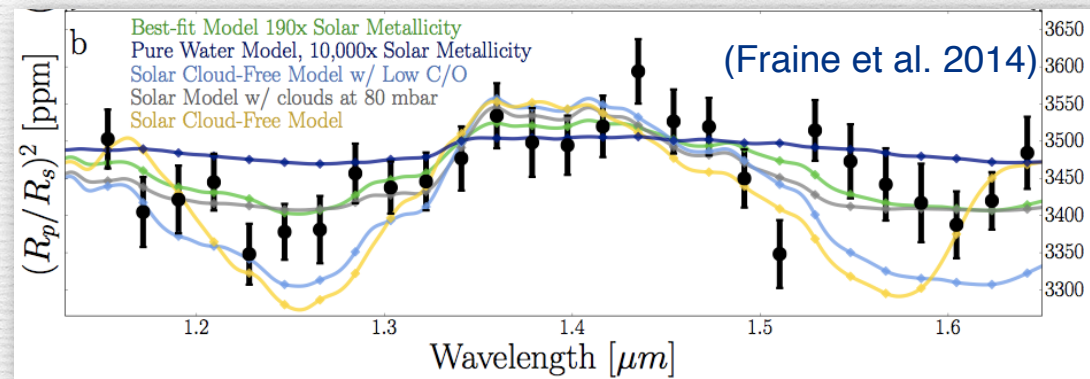
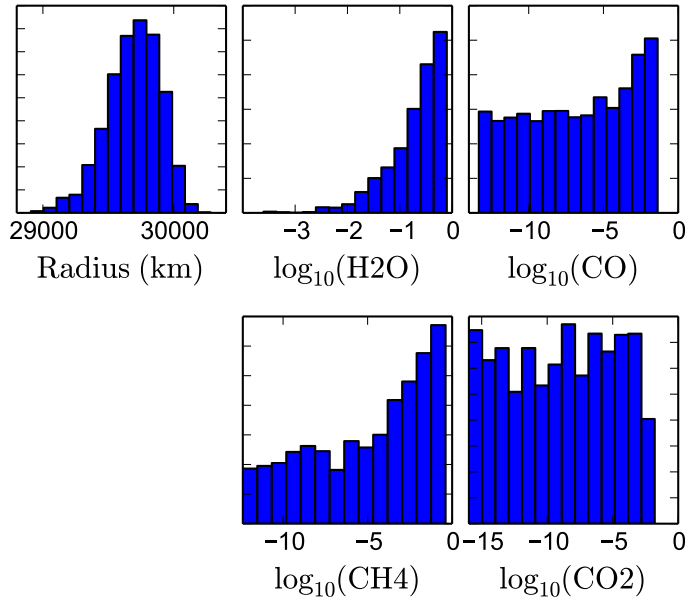
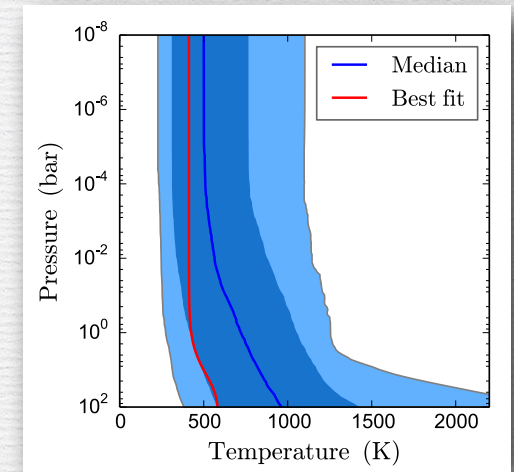


Replicate the work of  
Fraine et al. (2014).



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Replicate the work of Fraine et al. (2014).



# Conclusions:

- Models can be as complex as the data allow.
- Current exoplanet data is limited in wavelength coverage, wavelength resolution.

## Tools for exoplanet characterization:

- We developed an open-source, statistically-robust code to model exoplanet spectra and characterize planetary atmospheres.
- The retrieval analysis has been validated and can reproduce previous results on Neptune planet HAT-P-11b.

[github.com/exosports/BART](https://github.com/exosports/BART)

[github.com/pcubillos/MCcubed](https://github.com/pcubillos/MCcubed)

[github.com/exosports/transit](https://github.com/exosports/transit)

[github.com/dzesmin/TEA](https://github.com/dzesmin/TEA)

## Future development:

- Add Haze/Clouds and Rayleigh models.
- Add chemical disequilibrium: photo-chemistry, vertical mixing.
- Future instrument simulations: JWST & ELTs.